Computational Fluid Dynamics: Principles and Applications

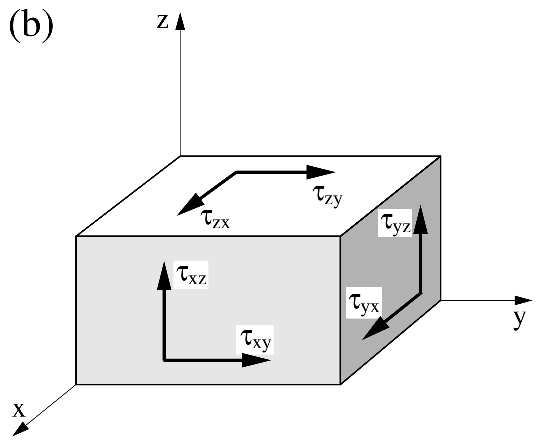
Jiri Blazek, http://www.cfd-ca.de

April 8, 2024

The following corrections apply to the 3rd edition of the book published in March 2015 by Elsevier Ltd. (ISBN: 978-0-08-099995-1).

Section 2.3: Viscous Stresses

Figure 2.3b on page 15 (stress components got interchanged):



Section 3.1.5: Central and Upwind Schemes

Text on page 42, initial paragraph:

... In this context, it is convenient to differentiate between the discretization of the convective and the viscous fluxes ($\vec{F}_c$ and $\vec{F}_v$ in Eq. (2.19), respectively). ...

Chapter 4: Structured Finite-Volume Schemes

Equation (4.64) on page 96:

$$M_R^- = \begin{cases} 0 & \text{if } M_R \geq +1 \\ -\dfrac{1}{4}(M_R - 1)^2 & \text{if } |M_R| < 1 \\ M_R & \text{if } M_R \leq -1 \, . \end{cases}$$

Chapter 5: Unstructured Finite-Volume Schemes

Equation (5.9) on page 128:

$$\vec{r}_c = \frac{\Omega_{123}\,\vec{r}_{c,123} + \Omega_{134}\,\vec{r}_{c,134}}{\Omega_{123} + \Omega_{134}}\,.$$

Equation (5.23) and text following on page 135:

$$\vec{n}_{01}\Delta S_{01} = \vec{n}_L \Delta S_L + \vec{n}_R \Delta S_R\,,$$

and the total face area is given by: $\Delta S_{01} = ||\vec{n}_L \Delta S_L + \vec{n}_R \Delta S_R||_2$.

Equation (5.55) on page 152:

$$\begin{bmatrix} \theta_1 \Delta x_{i1} & \theta_1 \Delta y_{i1} & \theta_1 \Delta z_{i1} \\ \theta_2 \Delta x_{i2} & \theta_2 \Delta y_{i2} & \theta_2 \Delta z_{i2} \\ \vdots & \vdots & \vdots \\ \theta_j \Delta x_{ij} & \theta_j \Delta y_{ij} & \theta_j \Delta z_{ij} \\ \vdots & \vdots & \vdots \\ \theta_{N_A} \Delta x_{iN_A} & \theta_{N_A} \Delta y_{iN_A} & \theta_{N_A} \Delta z_{iN_A} \end{bmatrix} \begin{bmatrix} \partial_x U \\ \partial_y U \\ \partial_z U \end{bmatrix}_i = \begin{bmatrix} \theta_1 (U_1 - U_i) \\ \theta_2 (U_2 - U_i) \\ \vdots \\ \theta_j (U_j - U_i) \\ \vdots \\ \theta_{N_A} (U_{N_A} - U_i) \end{bmatrix}$$

Equation (5.59) on page 153:

$$r_{11} = \sqrt{\sum_{j=1}^{N_A} (\theta_j \Delta x_{ij})^2}$$

$$r_{12} = \frac{1}{r_{11}} \sum_{j=1}^{N_A} \theta_j^2 \Delta x_{ij} \Delta y_{ij}$$

$$r_{22} = \sqrt{\sum_{j=1}^{N_A} (\theta_j \Delta y_{ij})^2 - r_{12}^2}$$

$$r_{13} = \frac{1}{r_{11}} \sum_{j=1}^{N_A} \theta_j^2 \Delta x_{ij} \Delta z_{ij}$$

$$r_{23} = \frac{1}{r_{22}} \left(\sum_{j=1}^{N_A} \theta_j^2 \Delta y_{ij} \Delta z_{ij} - \frac{r_{12}}{r_{11}} \sum_{j=1}^{N_A} \theta_j^2 \Delta x_{ij} \Delta z_{ij} \right)$$

$$r_{33} = \sqrt{\sum_{j=1}^{N_A} (\theta_j \Delta z_{ij})^2 - (r_{13}^2 + r_{23}^2)}\,.$$

Equation (5.62) on page 153:

$$\alpha_{ij,1} = \frac{\theta_j\, \Delta x_{ij}}{r_{11}^2}$$

$$\alpha_{ij,2} = \frac{\theta_j}{r_{22}^2} \left(\Delta y_{ij} - \frac{r_{12}}{r_{11}} \Delta x_{ij} \right)$$

$$\alpha_{ij,3} = \frac{\theta_j}{r_{33}^2} \left(\Delta z_{ij} - \frac{r_{23}}{r_{22}} \Delta y_{ij} + \beta \Delta x_{ij} \right)\,,$$

Equation (5.74) on page 162:

$$\left(\frac{\partial U}{\partial \ell}\right)_{IJ} \approx \frac{U_J - U_I}{\ell_{IJ}}\,,$$

Chapter 6: Temporal Discretization

Equation (6.12) on page 173:

$$\vec{W}_I^{(k)} = \vec{W}_I^{(0)} \;-\; \left[(\vec{R}_Q)_I^{(k-1)} \;-\; \Omega_I^n \vec{Q}_I^{(k-1)}\right] \left[\frac{\Omega_I^{n+1}}{\alpha_k \Delta t_I}\bar{I} - \left(\frac{\partial \vec{Q}}{\partial \vec{W}}\right)_I^{(k-1)}\right]^{-1}$$

Equation (6.19) on page 175:

$$\hat{\Lambda}_v^I = \max\left[\frac{4}{3\rho}(\mu_L + \mu_T),\; \frac{\gamma}{\rho}\left(\frac{\mu_L}{Pr_L} + \frac{\mu_T}{Pr_T}\right)\right]\frac{(\Delta S^I)^2}{\Omega}$$

and in similar way Eq. (6.21) on page 175 and Eq. (6.24) on page 176.

Reference [20] related to Eq. (6.22) on page 176:
Instead, the original reference is: Frink, N.T.; Parikh, P.; Pirzadeh, S.: *A Fast Upwind Solver for the Euler Equations on Three-Dimensional Unstructured Meshes*. AIAA Paper 91-0102, 1991.

Equation (6.59) on page 193:

$$\mathbf{D}\,\Delta\vec{W}_i^{(1)} = -\vec{R}_i^n - \sum_{j\in L(i)} \frac{1}{2}\left[(\Delta F_c^{(1)})_j \Delta S_{ij} - (r_A^*)_j \bar{I}\,\Delta\vec{W}_j^{(1)}\right]$$

$$\mathbf{D}\,\Delta\vec{W}_i^n \;= \mathbf{D}\,\Delta\vec{W}_i^{(1)} - \sum_{j\in U(i)} \frac{1}{2}\left[(\Delta F_c^n)_j \Delta S_{ij} - (r_A^*)_j \bar{I}\,\Delta\vec{W}_j^n\right]\,,$$

Equations (6.76), (6.77) and the related text on page 199:

$$\vec{R}_I^{(p+1)} \approx \vec{R}_I^{(p)} + \bar{J}_I\,\Delta\vec{W}^{(p)}$$

with $\Delta\vec{W}^{(p)} = \vec{W}^{(p+1)} - \vec{W}^{(p)}$ and $\bar{J} = \partial\vec{R}/\partial\vec{W}$ being the Jacobian matrix. Inserted into Eq. (6.75) while formulated as an iterative procedure, we obtain for the solution at stage k the expression

$$\left[\frac{(\Omega\bar{M})_I}{\Delta t} + a_{kk}\,\bar{J}_I\right]\Delta\vec{W}^{(p)} = \frac{(\Omega\bar{M})_I}{\Delta t}\left(\vec{W}_I^n - \vec{W}_I^{(p)}\right) - \sum_{l=1}^{k-1} a_{kl}\,\vec{R}_I^{(l)} - a_{kk}\,\vec{R}_I^{(p)}\,.$$

This is formally equivalent to Eq. (6.28) and can be solved by any of the implicit methods from the previous subsections. Upon convergence, the solution $\vec{W}^{(p+1)}$ of Eq. (6.77) approximates the intermediate solution at stage k from Eq. (6.75).

Equation (6.85) on page 204:

$$\vec{W}_I^{(k)} = \vec{W}_I^{(0)} - \frac{\alpha_k \Delta t_I^*}{\Omega_I^{n+1}}\left[\bar{I} + \frac{3}{2\,\Delta t}\alpha_k \Delta t_I^* \bar{M}^{n+1}\right]^{-1}$$

$$\cdot\left[\vec{R}_I(\vec{W}^{(k-1)}) + \frac{3}{2\,\Delta t}(\Omega\bar{M})_I^{n+1}\vec{W}_I^{(k-1)} - \vec{Q}_I^*\right]\,.$$

Chapter 7: Turbulence Modeling

Equation (7.4) on page 216:

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} - \frac{\partial v_i}{\partial x_j}\right) .$$

Equation (7.38) on page 226 and the text below it:

$$\tilde{S} = S + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2} ,$$

$$f_{v1} = \frac{\chi^3}{\chi^3 + C_{v1}^3} , \quad f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} ,$$

$$\chi = \frac{\tilde{\nu}}{\nu_L} ,$$

where S stands for the magnitude of the mean rotation rate, that is,

$$S = \sqrt{2\,\Omega_{ij}\Omega_{ij}} ,$$

and where Ω_{ij} is given by Eq. (7.4) . In order to avoid numerical difficulties, the term $\tilde{S}$ must never become zero or negative. One possibility is simple limiting like $\tilde{S} = \max(\tilde{S},\ 0.3\,S)$. More elaborate approaches were suggested in Ref. [49].

Equation (7.39) on page 226:

$$f_w = g\left(\frac{1 + C_{w3}^6}{g^6 + C_{w3}^6}\right)^{1/6} ,$$

$$g = r + C_{w2}(r^6 - r) , \quad r = \min\left[\frac{\tilde{\nu}}{\tilde{S}\,\kappa^2 d^2},\ 10\right] .$$

Equation (7.41) on page 227:

$$C_{b1} = 0.1355 , \quad C_{b2} = 0.622 ,$$

$$C_{v1} = 7.1 , \quad \sigma = 2/3 , \quad \kappa = 0.41 ,$$

$$C_{w1} = C_{b1}/\kappa^2 + (1 + C_{b2})/\sigma , \quad C_{w2} = 0.3 , \quad C_{w3} = 2 ,$$

$$C_{t1} = 1, \quad C_{t2} = 2, \quad C_{t3} = 1.2, \quad C_{t4} = 0.5 .$$

Text above Equation (7.42) on page 227:

As pointed out in Ref. [47] , it is convenient to substitute the diffusion term in Eq. (7.36) , that is,

$$\frac{1}{\sigma}\left\{\frac{\partial}{\partial x_j}\left[(\nu_L + \tilde{\nu})\frac{\partial \tilde{\nu}}{\partial x_j}\right] + C_{b2}\frac{\partial \tilde{\nu}}{\partial x_j}\frac{\partial \tilde{\nu}}{\partial x_j}\right\}$$

by the following expression ...

Text to **Initial and boundary conditions** on page 228:

The initial value of $\tilde{\nu}$ is usually taken as $\tilde{\nu} = 0.1\,\nu_L$. The same value is also specified at inflow or far-field boundaries. However, in cases where the transition term f_{t1} (and then mostly also f_{t2}) is omitted, that is, for fully turbulent flows, $\tilde{\nu}$ should be set to 3–5 times the value of ν_L at the inlet or far-field. At outflow boundaries, $\tilde{\nu}$ is simply extrapolated from the interior of the computational domain. At solid walls, it is appropriate to set $\tilde{\nu} = 0$ and hence $\mu_T = 0$.

References on page 246:

[49] Allmaras, S.R.; Johnson, F.T.; Spalart, P.R.: *Modifications and Clarifications for the Implementation of the Spalart-Allmaras Turbulence Model*. ICCFD7-1902, 7th Int. Conf. on Comput. Fluid Dynamics, 2012.

Chapter 8: Boundary Conditions

Equation (8.7) on page 257:

$$(\vec{F}_{c,w}\,\Delta S)_{i,2} = \begin{bmatrix} 0 \\ (n_x)_{i-1,2}\,(p_w)_{i-1/4} \\ (n_y)_{i-1,2}\,(p_w)_{i-1/4} \\ (n_z)_{i-1,2}\,(p_w)_{i-1/4} \\ 0 \end{bmatrix} \frac{\Delta S_{i-1,2}}{2} + \begin{bmatrix} 0 \\ (n_x)_{i,2}\,(p_w)_{i+1/4} \\ (n_y)_{i,2}\,(p_w)_{i+1/4} \\ (n_z)_{i,2}\,(p_w)_{i+1/4} \\ 0 \end{bmatrix} \frac{\Delta S_{i,2}}{2}\,.$$

Equation (8.8) and related text on page 258:

The pressures $(p_w)_{i-1/4}$ and $(p_w)_{i+1/4}$ in Eq. (8.7) can be obtained by linear interpolation, for example,

$$(p_w)_{i+1/4} = \frac{1}{4}(3p_{i,2} + p_{i+1,2})\,.$$

Equation (8.17) on page 261:

$$\rho_{i,2} = \frac{p_{i,3}}{T_w R} \quad \text{and} \quad (\rho E)_{i,2} = \frac{p_{i,3}}{\gamma - 1}\,.$$

Chapter 9: Acceleration Techniques

Point 5 of the solution steps described on page 311:

5. Multiply the residual by $\alpha_k\,\Delta t/\Omega$.

Equation (9.59) on page 313 (last term in the first column):

$$\bar{P} = \begin{bmatrix} \rho_p & 0 & 0 & 0 & \rho_T \\ \rho_p u & \rho & 0 & 0 & \rho_T u \\ \rho_p v & 0 & \rho & 0 & \rho_T v \\ \rho_p w & 0 & 0 & \rho & \rho_T w \\ \rho_p H + \rho h_p - 1 & \rho u & \rho v & \rho w & \rho_T H + \rho h_T \end{bmatrix}.$$

Text at the bottom of page 313:

where α_p and α_T are the compressibility coefficients at constant temperature and pressure, respectively ...

Text on page 318:

... the condition number Eq. (9.40) is reduced (the condition number is $C_N \approx 1.62$) and the ...

Chapter 10: Consistency, Accuracy and Stability

Equation (10.34) on page 349 (sign in front of the second term):

$$(D_x^I)_c\,\Delta U^n = \frac{\Lambda}{2}(\Delta U_{i+1}^n - \Delta U_{i-1}^n) - \Lambda\varepsilon^I(\Delta U_{i+1}^n - 2\Delta U_i^n + \Delta U_{i-1}^n)\,,$$

Equation (10.37) on page 350:

$$(D_x^I)_v\,\Delta U^n = \frac{\Lambda_v}{\Delta x}(\Delta U_{i+1}^n - 2\Delta U_i^n + \Delta U_{i-1}^n)\,.$$

Section 11.2.5: Assessment and Improvement of Grid Quality

Text on the top of page 388:

6. $(\text{volume})^4/$ (sum of squares of the areas of all triangular faces)^3 = $4.5725 \cdot 10^{-4}$.

A.11 TRANSFORMATION FROM CONSERVATIVE TO CHARACTERISTIC VARIABLES

Matrix of the left eigenvectors Eq. (A.86) on Page 435 (term in the 4th column on the 2nd row):

$$\bar{T}^{-1} = \begin{bmatrix} n_x a_5 - (n_z v - n_y w)\rho^{-1} & n_x a_1 u\, c^{-2} & n_x a_1 v\, c^{-2} + n_z \rho^{-1} \\ n_y a_5 - (n_x w - n_z u)\rho^{-1} & n_y a_1 u\, c^{-2} - n_z \rho^{-1} & n_y a_1 v\, c^{-2} \\ n_z a_5 - (n_y u - n_x v)\rho^{-1} & n_z a_1 u\, c^{-2} + n_y \rho^{-1} & n_z a_1 v\, c^{-2} - n_x \rho^{-1} \\ a_2(\varphi - cV) & -a_2(a_1 u - n_x c) & -a_2(a_1 v - n_y c) \\ a_2(\varphi + cV) & -a_2(a_1 u + n_x c) & -a_2(a_1 v + n_y c) \end{bmatrix}$$

$$\begin{bmatrix} n_x a_1 w\, c^{-2} - n_y \rho^{-1} & -n_x a_1 c^{-2} \\ n_y a_1 w\, c^{-2} + n_x \rho^{-1} & -n_y a_1 c^{-2} \\ n_z a_1 w\, c^{-2} & -n_z a_1 c^{-2} \\ -a_2(a_1 w - n_z c) & a_1 a_2 \\ -a_2(a_1 w + n_z c) & a_1 a_2 \end{bmatrix}.$$